FREE CONVECTIVE HEAT TRANSFER OF CYLINDERS OF LIMITED DIMENSIONS IN CYLINDRICAL HOUSINGS

A. S. Lyalikov and S. V. Solov'ev

UDC 536.253

Criterial equations are derived for the freely convective heat transfer of cylinders of limited dimensions eccentrically situated in cylindrical housings.

In order to calculate the thermal conditions of an element containing sources of heat evolution and situated inside a surrounding housing it is essential to possess information as to the heat transfer taking place between this element and the housing by natural convection. One of the several models of heat emitter/housing systems is that incorporating a cylindrical body of limited dimensions situated inside a cylindrical housing of finite dimensions and possessing internal heat sources (Fig. 1a); other characteristics of this model include symmetry relative to the plane x = 0, the displacement of the longitudinal axis of the inner cylinder in the vertical plane by an amount (±e), and the constancy of the surface temperatures of the cylinders t_1 and t_2.

An analytical or numerical solution of the system of energy, motion, and continuity equations constitutes an extremely complicated problem if we remember the three-dimensional nature of the fields of temperature, density, pressure, and vector-velocity components involved. The authors know of no publications presenting such results or attempting the solution of such a problem. An experimental investigation into the freely convective heat transfer of cylinders of limited dimensions in cylindrical housings was carried out in [1] in order to assess the dependence of freely convective heat transfer from the inner, heat-emitting body to the housing on the basic parameters of the system (eccentricity of the bodies, ratio of the dimensions, thermal conditions). It should be noted that the experimental data were not correlated in the best possible way: no well-based considerations were presented as to the way in which the generalization of the experimental data relating to such systems should be approached, while the criterial equations actually obtained failed to reflect the influence of the fundamental parameters on the heat-transfer process.

In this paper we shall present the results of a generalization of the experimental data of [1] regarding the freely convective heat transfer of cylinders of finite dimensions in cylindrical housings; the new approach has none of the shortcomings indicated.

1. An analysis of the conditions of similarity of the motion and heat-transfer processes associated with natural convection in the systems under consideration showed that, for the surface of the inner cylinder (after allowing for certain theoretical and experimental prerequisites), the average heat transfer should be represented by a relationship of the form

$$\bar{\mathrm{Nu}}_d = f\left(\mathrm{Ra}_d, \frac{d}{D}, \frac{l}{d}, \frac{L}{d}, \eta\right). \tag{1}$$

2. Figure 1b shows a typical relationship for the dimensionless surface temperature of the inner cylinder, expressed as a function of the dimensionless eccentricity η for several pairs of cylinders with different d/D, l/D, L/d ratios and constant power (for a specified pair of cylinders). As regards the manner in which θ varies with the displacement of the inner cylinder, we may distinguish three ranges of heat transfer:

a) that in which the inner cylinder is displaced within the limits

$$\pm\eta \leqslant 1 - \frac{12}{D-d} \tag{1a}$$

S. M. Kirov Tomsk Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 4, pp. 734-739, April, 1977. Original article submitted March 4, 1976.

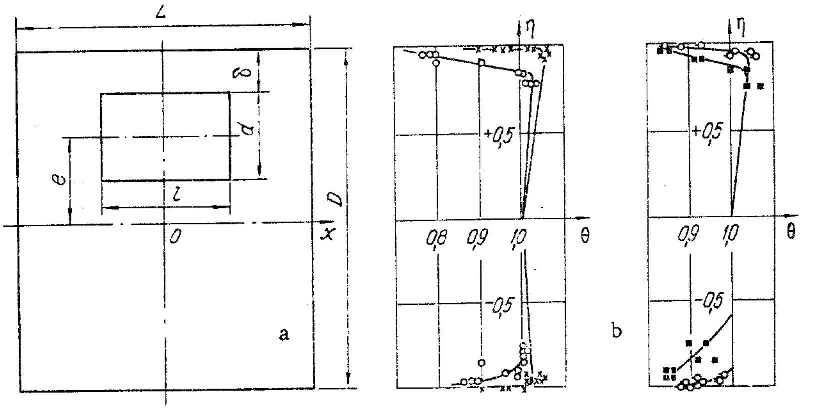

Fig. 1. Model of a cylinder of limited dimensions in a cylindrical housing (a) and the $\theta = f(\eta)$ relationship for several pairs of cylinders (b) (for notation, see Fig. 2).

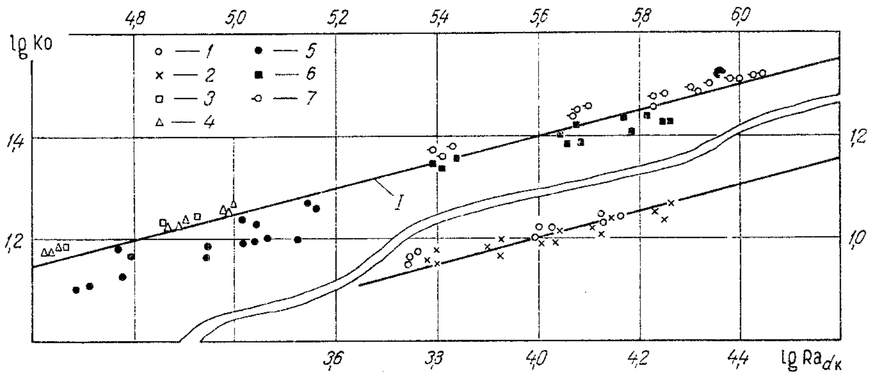

Fig. 2. Comparison between the correlating equations (2) and experimental data: I) according to Eq. (2); $Ko = \overline{Nu}_{dK}[0.468 \cdot (d/D)^{-0.46}(l/d)^{0.74}(L/d)^{-0.42}\Psi(\eta)]^{-1}$); 1) $d \times l = 12 \times 18$, $D \times L = 48 \times 48$; 2) 12×18, 200×200; 3) 24×24, 48×48; 4) 24×24, 96×72; 5) 24×24, 144×72; 6) 48×48, 96×96; 7) 48×48, 200×200 mm.

and the eccentricity exerts only a weak influence;

b) that in which the eccentricities are positive and close to the limiting value

$$1-\frac{12}{D-d} \leqslant \eta \leqslant 1-\frac{2}{D-d} \tag{1b}$$

(when the distance between the upper generators of the inner cylinder and the housing $\delta \approx 1$–6 mm); in this case, as the eccentricity increases there is a sharp fall in temperature, which (other experimental conditions being equal) signifies an increment in heat transfer;

c) that in which the eccentricities are negative and close to the limiting value

$$1-\frac{12}{D-d} \leqslant -\eta \leqslant 1-\frac{2}{D-d} \tag{1c}$$

(with $\delta \approx 1$–6 mm); here also there is a sharp fall in temperature with increasing eccentricity (by modulus).

In Eqs. (1a), (1b), and (1c), D and d are given in mm.

A generalization of the experimental heat-transfer data was carried out for each of these regions.

3. The experimental data for the region $\pm\eta \leqslant 1 - [12/(D - d)]$ were analyzed in generalized variables $\overline{Nu}_{dK} = \bar{\alpha} d/\lambda$ and $Ra_{dK} = g\beta\Delta t d^3/\nu^2$ and plotted on graphs of $\log \overline{Nu}_{dK} = f(\log Ra_{dK})$, which for purposes of greater clarity were constructed separately for positive, zero,

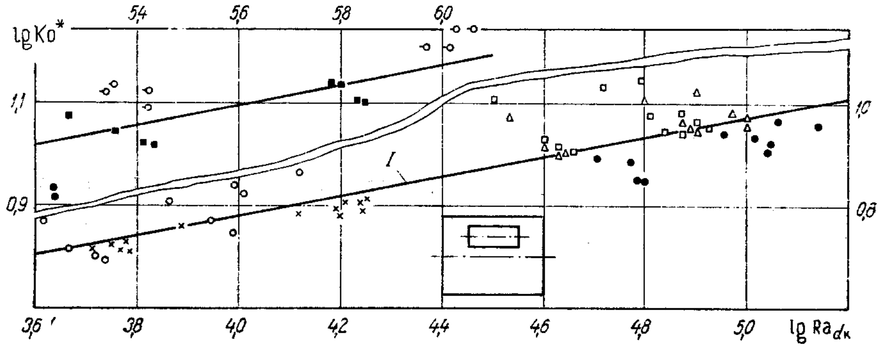

Fig. 3. Comparison between the correlating equation (3) and experimental data: I) according to Eq. (3); $Ko^* = \bar{N}u_{dK}[0.88 \cdot (d/D)^{-0.43}(l/d)^{1.03}(L/d)^{-0.57}(\delta/d)^{-0.1}]^{-1}$ (for notation, see Fig. 2).

and negative eccentricities. These graphs revealed that the heat-transfer processes for each pair of cylinders obeyed a law with an index of 0.25 attached to the Ra_{dK} number, i.e., $\bar{N}u_{dK} = cRa_{dK}^{0.25}$. Further analysis indicated that the coefficient c depended on the individual geometrical characteristics of each system:

$$\frac{d}{D},\ \frac{l}{d},\ \frac{L}{d},\ \eta.$$

The results of the generalization are represented by the criterial relationship

$$\bar{N}u_{d\text{K}} = 0.468\left(\frac{d}{D}\right)^{-0.46}\left(\frac{l}{d}\right)^{0.74}\left(\frac{L}{d}\right)^{-0.42}\psi(\eta)\,Ra_{d\text{K}}^{0.25};$$

$$0.06 \leqslant \frac{d}{D} \leqslant 0.5;\quad 2.0 \leqslant \frac{L}{d} \leqslant 16.6;$$

$$1.0 \leqslant \frac{l}{d} \leqslant 1.5;\quad 3.7 \leqslant \lg Ra_{d\text{K}} \leqslant 6.1; \tag{2}$$

$$\psi(\eta) = 10^{-0.05\eta}\ \text{for}\ \eta > 0;$$

$$\psi(\eta) = 1\ \text{for}\ \eta \leqslant 0.$$

Comparison between the experimental data (79 points) and Eq. (2) yields the results presented in Fig. 2. The deviation of the experimental points from Eq. (2) is no greater than +7 to −17.3%.

4. The generalization of the experimental data for the regions of positive and negative eccentricities close to the limiting values ($\delta \approx$ 1-6 mm) is also based on a relationship between the $\bar{N}u_{dK}$ and Ra_{dK} numbers, in which the power index of Ra_{dK} is first established from the log $\bar{N}u_{dK}$ = f(log Ra_{dK}) curves and the following coefficient is determined:

$$c = f\left(\frac{d}{D},\ \frac{l}{d},\ \frac{L}{d},\ \eta\right).$$

For each region we found the averaged values of the power index of Ra_{dK} from a series of experiments; it equaled 0.194 and 0.226, respectively, for the regions of limiting positive and negative eccentricities; the deviation of the power indices from 0.25 in the direction of lower values may be explained by the identical influence of the narrow gap between the inner and outer cylinders on the heat-transfer process in both cases. As a result of the correlation we obtained the following criterial relationships for the averaged heat transfer from the surface of the inner cylinder.

For the range of positive eccentricities close to the limiting value ($\delta \approx$ 1-6 mm)

$$\bar{N}u_{d\text{K}} = 0.88\left(\frac{d}{D}\right)^{-0.43}\left(\frac{l}{d}\right)^{1.03}\left(\frac{L}{d}\right)^{-0.57}\left(\frac{\delta}{d}\right)^{-0.1}Ra_{d\text{K}}^{0.194}. \tag{3}$$

A comparison between the experimental data (75 points) and Eq. (3) is presented in Fig. 3. The deviation of the experimental points from (3) is no greater than +23.0 to −18.3%.

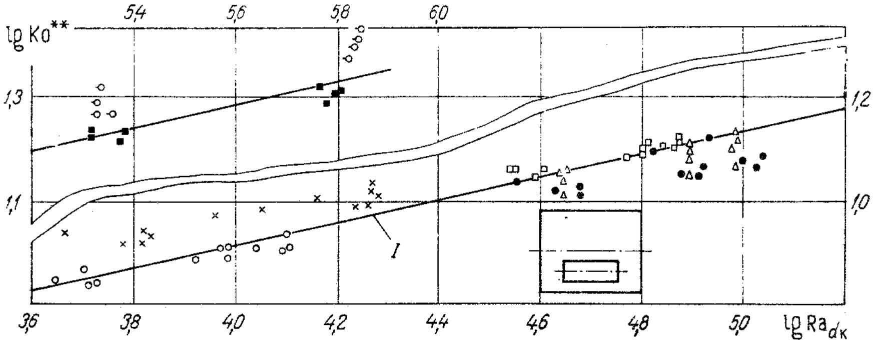

Fig. 4. Comparison between the correlating equation (4) and experimental data: I) according to Eq. (4); Ko** = $\bar{Nu}_{dK}\cdot[0.66(d/D)^{-0.43}(l/d)^{1.03}(L/d)^{-0.57}(\delta/d)^{-0.1}]^{-1}$ (for notation see Fig. 2).

For the range of negative eccentricities close to the limiting value ($\delta \approx$ 1-6 mm)

$$\bar{Nu}_{d\text{K}} = 0.66\left(\frac{d}{D}\right)^{-0.43}\left(\frac{l}{d}\right)^{1.03}\left(\frac{L}{d}\right)^{-0.57}\left(\frac{\delta}{d}\right)^{-0.1} Ra_{d\text{K}}^{0.226}. \tag{4}$$

A comparison between the experimental data (77 points) and Eq. (4) is presented in Fig. 4. The deviation of the experimental points from (4) lies in the range +17.8 to −16.1%.

For the regions of positive and negative eccentricities close to the limiting values the experimental data were analyzed by the method of least squares. Equation (3) provides an explanation for 86.5% of the general scatter in the experimental data relative to the average value of log Ko* = 0.846 [2]; the confidence interval for the power index of the Ra_{dK} number of Eq. (3) is 0.176; 0.212. The analogous characteristics for Eq. (4) are 91.3% relative to the average log Ko* = 0.993, confidence interval 0.210; 0.242. The confidence probability is taken as 95% in both cases.

The limits to the variations in d/D, l/d, L/d for Eqs. (3) and (4) are the same as in the case of Eq. (2).

In Eqs. (2)-(4) the defining dimension in the $\bar{Nu}_{dK}$ and Ra_{dK} numbers is the diameter d of the inner cylinder; the physical parameters of the air are referred to the average temperature of the outer cylinder (housing) t_K. In the Ra_{dK} number the quantity Δt is represented by the difference between the average surface temperatures of the inner and outer cylinders. In Eqs. (3) and (4) δ < 6 mm is the linear distance between the upper and lower generators of the cylinders for displacements of the inner cylinder close to the limiting values (upward or downward, respectively). In Eqs. (3) and (4), instead of the criterion η we have used the criterion δ/d, since this form of variable reveals the effect of δ on the process more directly than would be the case on using the eccentricity η, which is a function of two arguments

$$|\eta| = \frac{|e|}{\frac{1}{2}(D-d)} = \frac{|D/d-1-2\delta/d|}{D/d-1} = f\left(\frac{D}{d}, \frac{\delta}{d}\right).$$

5. The heat transferred by natural convection from the inner cylinder to the housing may be calculated by means of the equation

$$\theta_\text{K} = \bar{\alpha}(t_1 - t_2)F_1, \text{ W}. \tag{5}$$

We consider that the results here presented are of interest, both from the point of view of approaching the generalization of experimental data in the closed systems under consideration, and also from the point of view of the criterial heat-transfer equations so derived.

NOTATION

$\theta = (t_1 - t_2)_l/(t_1 - t_2)_0$, dimensionless temperature difference between the surfaces of the inner and outer cylinders; $(t_1 - t_2)_l$, $(t_1 - t_2)_0$, temperature differences on displacing the inner cylinder from the center of the outer and on retaining their coaxial disposition, respectively (for the same pair of cylinders, subject to the condition Q_k = idem); t_1, t_2, temperatures averaged over the surface of the inner and outer cylinders; $\pm\eta = \pm e/0.5(D - d)$, dimensionless eccentricity; $\bar{\alpha}$, heat-transfer coefficient averaged over the surface of the inner cylinder; $\bar{\alpha} = Q_K/(t_1 - t_2)F_1$; Q_K, heat transferred by convection from the inner to the outer cylinder; F_1, surface area of the inner cylinder.

LITERATURE CITED

1. P. A. Lyukshin and A. S. Lyalikov, Izv. Vyssh. Uchebn. Zaved., Probostr., 14, No. 11 (1971).
2. N. Draper and H. Smith, Applied Regression Analysis, Wiley (1966).

INFLUENCE OF NATURAL GAS EVOLUTION WHILE MELTING ON THE SEPARATION OF THE BOUNDARY LAYER

V. M. Stefanovskii

UDC 536.421.1:536.242

It is shown that under conditions of free convection the separation of air bubbles during the melting of ice and snow spheres displaces the point of separation of the boundary layer into the tail region. Relationships are derived for the separation angle as a function of the temperature difference and the porosity of the body.

The understanding and prediction of heat transfer on a melting surface is an extremely important problem of free convection. At the present time the mechanism of heat transfer in the course of melting is not entirely clear. One melting model proposed in [1, 2] started from the concept of the "injection" of melt into the boundary layer and predicted a fall in the heat-transfer coefficient in the course of melting.

We ourselves consider that no less obvious a process is the natural "injection" of gas into the boundary layer of the melt. A frozen solid in fact always contains bubbles. In the course of melting these complicate the hydrodynamic situation and undoubtedly have an effect on heat transfer. So far, however, no special attention has been paid to this. The situation is further complicated by the fact that, for example, in the freezing of water in ice molds the outer layer of ice is transparent and contains few gas inclusions. In order to avoid distortion of the sample shape, research workers have usually only studied the initial stages of melting. It is hardly suprising that according to the conclusions so drawn [3] air bubbles in ice have little effect on heat transfer.

Since heat transfer is governed by the hydrodynamic situation, we shall here consider the special problem as to the influence of bubbles evolved in the course of melting on the separation of the boundary layer.

We made our samples in a spherical demountable mold with an internal diameter of 147 mm. In order to transport the sample and attach a weight to it during immersion we froze a Nichrome wire into the ice. Before the experiment the mold was held in melting ice. After elimination of the supercooling, the sphere was easily extracted from the ice mold and weighed and measured at zero temperature (in a refrigerating chamber). Melting was carried out under conditions of free convection at a constant temperature in a tank with a capacity of 100 liters. In order to visualize the convective flow, aluminum particles were added to the water and illuminated with a flat light beam. At fixed time intervals the profile of the body and the flow of melt were successively photographed.

These observations showed that at $t < 5°C$ there was a rising, unseparated flow; at $t > 5°C$ the flow was downward and the ice sphere behaved as a body of a shape not readily admitting flow around it, the boundary layer undergoing separation. Thereupon the sphere changed

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 4, pp. 740-742, April, 1977. Original article submitted February 4, 1976.